AD-A089 427 MASSACHUSETTS INST OF TECH CAMBRIDGE LAB FOR INFORMA--ETC F/G 12/1
STABILITY AND CONVERGENCE OF ADAPTIVE CONTROL ALGORITHMS: A SUR--ETC(U)
MAY 80 L S VALAVANI AFOSR-77-3281
UNCLASSIFIED LIDS-P-989 AFOSR-TR-80-0778 NL

1 OF 1
AD
A089427

END
DATE
FILMED
10-80
DTIC

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER AFOSR-TR-80-0778	2. GOVT ACCESSION NO. AD-A089 427
3. RECIPIENT'S CATALOG NUMBER	
4. TITLE (and Subtitle) STABILITY AND CONVERGENCE OF ADAPTIVE CONTROL ALGORITHMS: A SURVEY AND SOME NEW RESULTS	5. TYPE OF REPORT & PERIOD COVERED Interim rept.
	6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Lena S. Valavani	8. CONTRACT OR GRANT NUMBER(s) AFOSR-77-3281
9. PERFORMING ORGANIZATION NAME AND ADDRESS M. I. T. Lab. for Information & Decision Systems Cambridge, MA 02139	10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A1
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research/NM Bolling AFB, Washington, DC 20332	12. REPORT DATE May 1980
	13. NUMBER OF PAGES 11
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) LIDS-P-999	15. SECURITY CLASS. (of this report) UNCLASSIFIED
	15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

SEP 24 1980

A

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Although adaptive controllers have been designed for a number of years, the central question of global asymptotic stability of the overall feedback adaptive loop and its associated error equations remained open until recently. The two main approaches used to design controllers from a stability viewpoint - Lyapunov's Direct Method and Popov's Hyperstability Theory - only assure boundedness of the (augmented) state and parameter errors. The difficulties encountered in both approaches are analyzed and shown to be exactly analogous.

(OVER)

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED 411950

AD A089427

DDC FILE COPY

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

20. Abstract cont.

Subsequently, a generic model for an adaptive controller is proposed from which already existing adaptive algorithms can be derived with minor modifications. The model unifies various algorithms, under the essential requirement for positive reality of the associated error transfer function, and its proof of stability contains others suggested to date as special cases. Finally some general comments are made regarding rates of convergence, performance of the algorithms in the presence of disturbances and unmodeled dynamics, ease of implementation, etc.

A

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE(When Data Entered)

AFOSR-TR- 80-0778

May, 1980

LIDS-P-989

STABILITY AND CONVERGENCE OF ADAPTIVE CONTROL ALGORITHMS: A SURVEY AND SOME NEW RESULTS.*

Lena S. Valavani

L.I.D.S., Massachusetts Institute of Technology

ABSTRACT

Although adaptive controllers have been designed for a number of years, the central question of global asymptotic stability of the overall feedback adaptive loop and its associated error equations remained open until recently. The two main approaches used to design controllers from a stability viewpoint - Lyapunov's Direct Method and Popov's Hyperstability Theory - only assure boundedness of the (augmented) state and parameter errors. The difficulties encountered in both approaches are analyzed and shown to be exactly analogous.

Subsequently, a generic model for an adaptive controller is proposed from which already existing adaptive algorithms can be derived with minor modifications. The model unifies various algorithms, under the essential requirement for positive reality of the associated error transfer function, and its proof of stability contains others suggested to date as special cases. Finally some general comments are made regarding rates of convergence, performance of the algorithms in the presense of disturbances and unmodeled dynamics, ease of implementation, etc..

I INTRODUCTION

Adaptive control has witnessed some very important theoretical developments in recent years. Although the state of the art is still far from being satisfactory for most practical applications, the progress made so far is encouraging and suggestive of new directions for the improved performance of adaptive algorithms.

In the past few years various adaptive controllers were proposed reflecting different philosophical viewpoints, such as direct and indirect control, and based on different methodologies, motivated either from a stability approach to the problem or from optimization considerations. Quite recently, the mathematical equivalence between direct and indirect control was established [1,2], - at least for certain parameterizations of the plant - while it has become increasingly evident that there is considerable overlap of the problems encountered using either the optimization or the stability approach for designing adaptive controllers with inaccessible process states, particularly as the latter relate to the question of global asymptotic stability of the resulting feedback adaptive loop. In fact, this question became of crucial importance in the study of adaptive systems and was not resolved until very recently [3,4,5]. Even so, the results pertain only to the case of "deterministic" adaptive systems[1] while no proof of the stability in the global sense is available for "stochastic" adaptive schemes, where disturbances are present in the form of observation and/or process noise. The most rigorous results along these lines were obtained by Ljung [6,7], who used the (by now) well-known method of the "associated differential equation" corresponding to the stochastic adaptive problem. However, these proofs are valid only locally.

The two major aspects of the general control problem were, first, the design of a controller for an unknown plant from input-output data alone, with no accessibility to any other points in the plant; and second, because of noise considerations, the exclusion of explicit use of differentiators. Since adjustment of the parameters is not possible due to restricted accessibility of the plant, such a controller would have to be realized by synthesis of its input. This difficulty was enhanced by the generation of nonlinear time-varying differential (difference) equations for the adaptive loop, due to feedback. Thus, the adaptive control problem turned out to be considerably more complex than that of the adaptive observer,[2] where not only every point in the adaptive system (observer) was accessible but also its open-loop scheme resulted in linear differential equations. The first clear exposition of all complications arising in the adaptive control problem was given in [8].

From the mathematical point of view, the inaccessibility of any point in the plant other than its input and output could be translated as the difficulty in realizing a positive real output error transfer function (Ref. Section III). An important contribution along these lines was made by Monopoli [9], who suggested an ingenious scheme for control,

* This research was supported in part by the Air Force Office of Scientific Research under contract number [redacted].AFOSR-77-3281

1 We note, here, that our ensuing discussion will be primarily concerned with the on-line control of linear, time-invariant, SISO plants under insufficient knowledge of their parameters and/or state variables. There are very few results - if any at all - that are rigorous pertaining to the more general cases of time-varying and/or multivariable plants.

2 This problem was well understood and completely resolved by the first half of the decade.

Approved for public release; distribution unlimited.

80 9 22 260

AIR FORCE OFFICE OF SCIENTIFIC RESEARCH (AFSC)
NOTICE OF TRANSMITTAL TO DDC
This technical report has been reviewed and is
approved for public release IAW AFR 190-12 (7b).
Distribution is unlimited.
A. D. BLOSE
Technical Information Officer

by making use of an augmented error signal.[3] However, while error augmentation maintained boundedness of the state and parameter errors, it could no longer be assumed that either the plant or the model states remained bounded (Ref. Section II). Motivated by Monopoli's approach, Narendra and Valavani [10] succeeded in designing a simple controller structure which realized a positive real output error transfer function during adaptation. The stability problem of the overall adaptive loop was also clarified in that work and its solution was presented in the form of a conjecture. A modified version of the conjecture involving an additional feedback term was recently verified [3]. At about the same time, Feuer and Morse [11] suggested an alternative controller but its extreme complexity precludes its use in practical applications. Since then, several authors [5,2,12,13] have also attempted this problem for both discrete and continuous systems. In [2] both discrete as well as continuous-time controllers are discussed and compared. In [12] and [13] the continuous adaptive problem is treated - using indirect control ideas - when the input to the system is "sufficiently rich." Proof of asymptotic stability here heavily hinges on the assumption of convergence of the parameters. However, simulation studies of any of the adaptive control algorithms available to date, consistently show that the above assumption almost never holds, despite the fact that the output errors tend to zero; this fact should not be surprising, since the control objective - i.e., output error tending to zero - is nonetheless satisfied.

Finally, in [5] the authors present three very simple adaptive controllers for discrete time systems using both a direct as well as an indirect control approach. The motivation for the schemes they propose comes from optimization considerations rather than the explicit use of the stability approach in the design, since two of their algorithms are projection algorithms on the (estimated) parameter space and a third algorithm employs recursive least squares. After careful study of the algorithms in [5] and contrary to what was initially believed, it is interesting to find out that they also implicitly realize a positive real output error transfer function. This is an important observation, since positive reality has been a key requirement in the analysis and design of adaptive controllers using either Lyapunov's Direct Method or Popov's Hyperstability Theory. With either method, adaptive laws that guarantee the global stability of the overall scheme are derived. Unfortunately, however, neither one alone can allow the conclusion of asymptotic stability for all cases of control and, therefore, special analysis is further required.

In Section II we discuss an error model which is most frequently (almost invariably) used in adaptive control under limited state information. In Section III we present briefly the main features of both Hyperstability Theory and Lyapunov's method and point out the difficulties encountered in the proofs of asymptotic stability of augmented error systems. Section IV contains the main contribution of this paper which is a generic model for the design of stable adaptive controllers. This model captures in a unified manner all the essential features of already existing adaptive control algorithms. The latter can be derived from this general model with minor modifications. This is discussed in Section V for a representative number of algorithms. Moreover, the model clarifies the role of relative degree and positive reality of the error transfer function and shows quite convincingly that the latter is one of the two main underlying factors in the already existing asymptotically stable adaptive schemes; the second is that the parameter adjustment law satisfy some conditions such as being an L^2 function, which is the only case for which a proof is available at present. The asymptotic stability proof is presented briefly - rather outlined - in Section VI. In retrospect, some of the stability proofs given to date are seen to be special cases of the (outline of) proof in VI. Section VII contains general comments on adaptive algorithms, i.e., regarding their rates of convergence, their behavior in the presence of disturbances and unmodeled system dynamics, etc. The conclusion is given in Section VII.

Due to space limitations, presentation of the results is kept concise and proofs are only outlined in most cases instead of given in detail. Whenever necessary, reference is made either to already existing proofs or to a more expanded - hence complete - version of this paper (Sections IV, V and VI in particular) in which the proofs are included. The discussion is carried out for continuous time systems and the easier cases of discrete-time systems are mentioned whenever appropriate. (It is much easier to derive results going from continuous to discrete-time rather than vice versa).

II AN ERROR MODEL IN ADAPTIVE CONTROL

The error model described in this section is also known as Prototype III in the adaptive control literature [14]. It applies to the case where only the plant output is accessible and, therefore, neither the entire error nor plant state vectors can be used in the implementation of the adaptive algorithm. Stability of such algorithms is then difficult to ensure, unless the adaptive system can be parameterized in a specific form. It is quite often the case that the error differential equations of adaptive systems whose entire state vector is not accessible can be put in the following form [14]:

$$\dot{e}(t) = Ae(t) + b\phi^T(t)\, u(t) \qquad (1)$$

$$e_1(t) = c^T e(t)$$

where $e(t)$ is an (nxl) state vector, $\phi(t)$ and $u(t)$ are m-dimensional vectors with the elements of $u(t)$ piecewise continuous and uniformly bounded. A is a stable (nxn) matrix, b and c are (nxl) constant vectors, with (A,b) completely controllable, and the transfer function $c^T(sI-A)^{-1}b$ is strictly positive real (SPR). The elements of the vector $\phi(t)$ are unknown but the time derivative $\dot{\phi}(t)$ can be adjusted using the signals $u(t)$ and $e_1(t)$ which can be measured. The aim of this adjustment is to make $\lim_{t\to\infty} e_1(t) = 0$.

If the adaptive law

$$\dot{\phi}(t) = -\Gamma u(t)e_1(t) \qquad \Gamma = \Gamma^T > 0 \qquad (2)$$

[3] Use of an "augmented error" became necessary whenever the relative degree of the plant transfer function (i.e., excess of poles over zeros) was greater than or equal to three.

is chosen, it can be shown easily using the Kalman-Yakubovich Lemma that

(i) $e(t)$ and $\phi(t)$ are bounded

(ii) $\lim_{t \to \infty} e(t) = 0$

and (iii) if $u(t)$ is "sufficiently rich" [15]

$$\lim_{t \to \infty} \phi(t) = 0$$

Proof: Choosing $V(e,\phi) = 1/2\, [e^T Pe + \phi^T \Gamma^{-1} \phi] > 0$ as a Lyapunov function candidate, the time derivative $\dot{V}(e,\phi)$ can be written as

$$\dot{V}(e,\phi) = 1/2 e^T [A^T P + PA] e + e^T Pb\phi^T u + \phi^T \Gamma^{-1} \dot{\phi}$$

By the Kalman-Yakubovich Lemma, a matrix $P = P^T > 0$ exists such that

$$A^T P + PA = -qq^T - \varepsilon L$$

$$Pb = c$$

for some vector q, matrix $L = L^T > 0$, and $\varepsilon > 0$ iff $c^T(sI - A)^{-1}b$ is strictly positive real (SPR). In such a case,

$$\dot{V}(e,\phi) = -1/2 e^T (qq^T + \varepsilon L) e + e_1 \phi^T u + \phi^T \Gamma^{-1} \dot{\phi}$$

By choosing the adaptive law according to (2), $\dot{V}$ becomes

$$\dot{V}(e,\phi) = -1/2 e^T (qq^T + \varepsilon L) e \leq 0$$

so that the system is stable and (i) holds. V, furthermore, is a nonincreasing function of time which is bounded from below and hence converges to a finite value V_∞.

We first consider the case where the input $u(t)$ is uniformly bounded. In such a case we have

$$\lim_{t \to \infty} \int_0^t \dot{V}(t) dt = V_\infty - V(0)$$

which is a finite number and $\dot{V}(t)$ is uniformly continuous since $\dot{e}(t)$ and hence $\ddot{V}(t)$ are bounded. Hence by a well known lemma [18]

$$\lim_{t \to \infty} \dot{V}(t) = \lim_{t \to \infty} -1/2\, e^T (qq^T + \varepsilon L) e(t) = 0$$

or $\lim_{t \to \infty} e(t) = 0$ and $\lim_{t \to \infty} e_1(t) = 0$

By (2), since $u(t)$ is bounded, it follows that

$$\lim_{t \to \infty} \dot{\phi}(t) = 0$$

It has also been shown [15] that if $u(t)$ is "sufficiently rich,"

$$\lim_{t \to \infty} \phi(t) = 0$$

The results stated so far may now be summarized as follows:

a. If $V(e,\phi)$ is positive definite and $\dot{V}(e,\phi)$ is negative semidefinite, $e(t)$, $\phi(t)$ are bounded if $e(0)$, $\phi(0)$ are bounded.

b. If the input $u(t)$ is bounded, $\ddot{V}(e,\phi)$ is also bounded and hence

$$\lim_{t \to \infty} e(t) = 0 \qquad \lim_{t \to \infty} \dot{\phi}(t) = 0$$

c. If $u(t)$ is bounded and "sufficiently rich"

$$\lim_{t \to \infty} \phi(t) = 0$$

The problem becomes considerably more complicated when any finite time truncation of $u(t)$ is bounded, i.e., $u(\cdot) \epsilon L_e^\infty$ but $u(t)$ is not uniformly bounded. It is now no longer possible to conclude from the above analysis that $e(t)$ and $\dot{\phi}(t)$ behave as in b. However, it was recently shown [16] that even when the inputs are unbounded, the error system can be uniformly asymptotically stable provided the inputs are "uniformly exciting" in the sense of [16]. Some of the principal difficulties in the resolution of the adaptive control problem have been related to these questions that arise when the control input $u(t)$ to the plant cannot be assumed bounded and, further "uniform excitedness" is not assured. Hence an alternate route had to be taken in the proof.

A. A Modified Error Model for Use in Asymptotic Stability Proofs in "Augmented Error" Systems.

Perhaps the most important step in achieving asymptotic stability of the adaptive system in [10] was in introducing an extra feedback term in the error model described by equation (1). Figure 1 is a schematic representation of the modified error model.

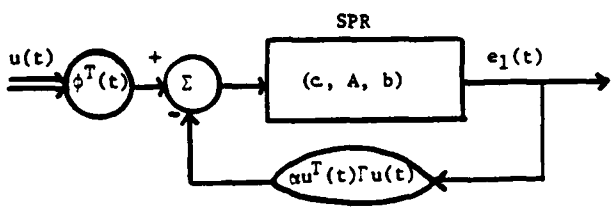

FIGURE 1

The state equations describing the above system are given by:

$$\dot{e}(t) = Ae(t) + b[\phi^T(t)u(t) - \alpha u^T(t)\Gamma u(t) e_1(t)]$$

$$\alpha > 0, \quad \Gamma = \Gamma^T > 0 \tag{3}$$

$$e_1(t) = c^T e(t)$$

and the adaptive law is given as before by

$$\dot{\phi}(t) = -\Gamma u(t) e_1(t) \qquad \Gamma = \Gamma > 0 \tag{4}$$

Choosing as a Lyapunov function candidate the same $V(e,\phi)$ as before, we obtain its time-derivative according to equations (3) and (4) as

$$\dot{V}(e,\phi) = 1/2 e^T(t)[A^T P + PA]e(t) + e^T(t)Pb[\phi^T(t)u(t) - \alpha u^T(t)\Gamma u(t)\, e_1(t)] + \phi^T(t)\Gamma^{-1}\dot{\phi}(t).$$

Using the K-Y Lemma and equation (4)

$$\dot{V}(e,\phi) = -1/2e^T(qq^T + \varepsilon L)e - \dot{\phi}^T\Gamma^{-1}\dot{\phi} \tag{5}$$

From the fact that $V(e,\phi)$ is bounded for bounded initial values of e and ϕ and from (5) we can conclude that (i)$e \in L^2$, $\dot{\phi} \in L^2$. The fact that $\dot{\phi} \in L^2$ is crucial in the stability proof of Section VI. The rest of the stability arguments follow exactly as in the first part of this section.

III LYAPUNOV'S METHOD AND HYPERSTABILITY THEORY IN ADAPTIVE CONTROL

Lyapunov's Direct Method and, relatively more recently, Popov's Hyperstability Theory have provided the principal framework for the analysis and design of adaptive systems using a stability approach. Using this approach, adaptive problems are formulated as stability problems of multivariable nonlinear nonautonomous systems. When Lyapunov's method is employed, the asymptotic stability of a set of error differential equations is studied using a suitable choice of a Lyapunov function candidate (as in Section II). While the theory has been applied effectively to autonomous systems, it has been less decisive in the adaptive context where the equations are nonautonomous. The difficulty lies in the fact that always the choice of the adaptive law for the updating of the parameters is such that the time derivative of the Lyapunov function is invariably negative semidefinite, rather than definite. This together with the fact that in the more general case of time-varying (rather than autonomous) differential equations, which arise in adaptive control, a correspondence between their limiting sets and invariant sets has not been established, precludes use of LaSalle's theorem for proof of asymptotic stability.

The hyperstability approach [17-20] which has been increasingly used as an alternative to Lyapunov's method, requires the problem to be recast as the stability of a feedback loop with a linear time-invariant operator in the forward path and a passive operator in the feedback path. This structure provides the designer with greater flexibility in choosing the adaptive laws, since it is merely required to make the feedback block satisfy some passivity conditions for the system to be hyperstable. However, as will be seen in the sequel, this approach too meets with the same analytical difficulties when unbounded signals are present.

Since the stability of the error model in Section II was analyzed using Lyapunov's method, we will present in this section only the hyperstability approach. Consider now a completely controllable and completely observable system B_1.

$$\left.\begin{aligned} \dot{x}(t) &= Ax(t) + bu(t) \\ y(t) &= c^T x(t) \end{aligned}\right\} \text{ System } B_1$$

where A is an (nxn) matrix, x(t) an (nx1) vector, u(t) a piecewise continuous input and y(t) the output.

Hyperstability of B_1 is then defined by the property which requires that the state x(t) be bounded for a certain class of inputs u(t). This class is defined by those $u(\cdot)$ which satisfy for all T

$$\int_0^T u(t)y(t)dt \le \delta[\|x(0)\|]\sup_{0 \le t \le T}\|x(t)\|$$

where δ is a positive constant. For the purposes of our discussion it is adequate to limit ourselves to the class of inputs which satisfy

$$\int_0^T u(t)y(t) \le l^2 \quad \text{independent of } T \tag{6}$$

as suggested by Landau [19], where l is an arbitrary constant, independent of T.

Definition: The system B_1 is hyperstable with respect to any u(t) which satisfies (6) if there exists a positive constant k such that

$$\|x(t)\| \le k[\|x(0)\| + l] \quad \text{for all } t \tag{7}$$

Definition: The system B_1 is said to be asymptotically hyperstable with respect to any u(t) satisfying (6) which is also bounded if the inequality (7) holds together with

$$\lim_{t \to \infty} x(t) = 0 \tag{8}$$

The main theorems of Popov may now be stated as follows.

Theorem 1. (Hyperstability): A necessary and sufficient condition for the system B_1 to be hyperstable is that the transfer function

$$W(s) = c^T(sI-A)^{-1}b \tag{9}$$

be positive real (PR).

Proof: See [17].

Theorem 2. (Asymptotic Hyperstability): A necessary and sufficient condition for the system B_1 to be asymptotically hyperstable is that $W(s)$ in (9) be strictly positive real. (SPR).

If B_1 is SPR, it can be shown [18] that a positive definite function $\rho(x)$ exists such that

$$\int_0^T \rho(x)dx \le \int_0^T u(t)y(t)dt \tag{10}$$

When u(t) is bounded, the function $\rho[x(t)]$ can also be shown to be uniformly continuous. Hence, from (10) and (6) and a well known lemma (Barbălat's Lemma in [18]), it follows that

$$\lim_{t \to \infty} \rho[x(t)] = 0 \quad \text{or} \quad \lim_{t \to \infty} x(t) = 0$$

Remark: Uniform continuity of $\rho[x(t)]$ [and of $\dot{V}(x,t)$ analogously], is only a sufficient condition in these proofs.

We now apply the above theorems to the error model discussed in Section II. Figure (2) is a schematic representation of system (1) and (2) in the form suggested in theorems 1 and 2.

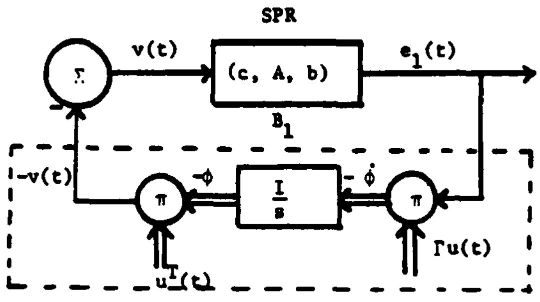

FIGURE 2 B_2

Since $\int_0^T v(t)e_1(t)dt = -\int_0^T \phi^T(t)\Gamma^{-1}\dot{\phi}(t)dt =$

$$\frac{\|\phi(0)\|^2_{\Gamma^{-1}} - \|\phi(T)\|^2_{\Gamma^{-1}}}{2} \leq \frac{\|\phi(0)\|^2_{\Gamma^{-1}}}{2}$$

by Theorem 2 the state vector e(t) of B_1 is bounded. Similarly, by considering B_2 in which 1/s is a positive real transfer function, the state vector $\phi(t)$ can be shown to be bounded. This corresponds to condition (i) in Section II. If u(t) is bounded, then so is the input u(t) to B_1. Since B_1 has a SPR transfer function, by Theorem 2, e(t) and hence $e_1(t)$ tend to zero as $t \to \infty$. This corresponds to condition (ii) in the previous section. The rest of the analysis is identical to that in Section II.

Note: In many adaptive control situations it may be easier to check conditions (6) and (10), rather than searching for a Lyapunov function and trying to assure at least negative semidefiniteness of its time derivative. In fact, the integral conditions (6) and (10) may provide the designer with more flexibility in choosing the adaptive laws, rather than the pointwise condition required of $\dot{V}(e,t)$. It is obvious from the discussion that hyperstability theory offers design advantages rather than analytical ones.

IV A GENERIC MODEL FOR ADAPTIVE CONTROLLER STRUCTURES

Before we proceed to describe the general controller model, we give a brief statement of the problem below.

A. Statement of the Problem

The plant P to be controlled is completely represented by the input-output pair $[u(t), y_p(t)]$ and can be modeled by a linear time-invariant system

$$\dot{x}_p = A_p x_p + b_p u(t)$$
$$y_p = h_p^T x_p(t)$$

where A_p, b_p, h_p are a matrix and vectors of constants and are of compatible dimensions and such that the transfer function

$$W_p(s) = h_p^T(sI - A_p)^{-1}b_p \triangleq \frac{k_p p(s)}{q(s)} \qquad (11)$$

is strictly proper; p(s) is a monic Hurwitz polynomial of degree $m(\leq n-1)$, q(s) a monic polynomial of degree n and k_p a constant gain parameter. We further assume that only m, n and the sign of k_p are known for use in the design of an adaptive controller and only one output of the plant, i.e., $[h_p^T = (1,0...0)]$ is accessible for measurement. $n^* = (n-m)$ is referred to as the relative degree of the plant and plays an important role in the parametrization of the controller in this section. We note here, that since the only points of access to the plant are its input and output, any internal structure representation (h_p, A_p, b_p) can be assumed which satifies (11). However, the "specific parametrization" chosen is an essential ingredient in the subsequent development of the controller-reference model structure.

A reference model M represents the behavior desired from the plant when it is augmented with a suitable controller. The model has a reference input r(t) which is uniformly bounded and an output $y_m(t)$. The transfer function of the model, denoted by $W_m(s)$ may be represented as

$$W_m(s) \triangleq k_m \frac{p_m(s)}{q_m(s)}$$

where $p_m(s)$ is a monic Hurwitz polynomial of degree $m_1 \leq m$,[4] $q_m(s)$ is a monic Hurwitz polynomial of degree n and k_m is a constant gain. $W_m(s)$ is completely specified and the aim of the design is to generate suitable bounded inputs u(t) to the plant, so that the deviation of the plant from the desired behavior as measured by an error signal $\varepsilon_1(t)$ where

$$|\varepsilon_1(t)| \triangleq |y_p(t) - y_m(t)|$$

tends to zero as $t \to \infty$.

B. Plant and Controller Representation.

Here, we present a general observer-controller structure which yields a simple relation between observer and controller parameters (similarly as in [1]). In fact, since the mathematical equivalence between direct and indirect control was proven [1,2], the general structure presented here could be viewed as either. The following Lemma and Corollaries are needed to justify what will follow.

Lemma 1: Given the relatively prime polynomials p and q of degrees $m(\leq n-1)$ and n respectively, with q monic, a monic polynomial Γ and a polynomial Δ of degree n exist such that the polynomial

$$\Gamma q + \Delta p = M$$

where M is any 2n degree monic polynonial.
Pf: Given in [10].

Corollary 1: If M_1 is any monic polynomial of degree (2n+m) such that $M_1 = MC$, where C is a monic polynomial of degree m, polynomials Γ_1 and Δ_1 of degrees (n+m) exist which satisfy

[4] Since the controller structure to be described in the sequel is motivated by the indirect control representation in [14], we will assume, with no loss of generality, that $m_1 = m$. When $m_1 > m$, r(t) has to be suitably prefiltered.

THIS PAGE ... PRACTICABLE FROM COPY ...

$$\Gamma_1 q + \Delta_1 p = M_1$$

This follows directly from Lemma 1 by choosing $\Gamma_1 = \Gamma\sigma$ and $\Delta_1 = \Delta\sigma$

Corollary 2: Given two polynomials $\bar{L}(s)$ and $\Lambda(s)$ of degrees $(n-m)$ and n respectively, if $p(s)$ and $q(s)$ are relatively prime polynomials of degrees m and n, then the rational function $p(s)/q(s)$ can be expressed as

$$\frac{p(s)}{q(s)} = \frac{\Gamma(s)}{\Lambda(s)\bar{L}(s) + \Delta(s)} \qquad (12)$$

by the proper choice of the polynomials $\Gamma(s)$ and $\Delta(s)$ of degree n.

Proof: Equation (12) implies $\Gamma q - \Delta p = p\Lambda\bar{L}$ and since the RHS is of degree $2n$, Lemma 1 applies directly.

Figure 3b is an equivalent nonminimal representation of a plant with an irreducible transfer function $p(s)/q(s)$, according to Corollary 2 and with $\Lambda(s) = p_m(s)\bar{L}(s)$.

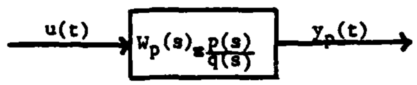

FIGURE 3a

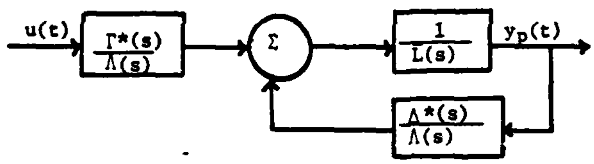

FIGURE 3b

In figure 3 the only condition on $\bar{L}(s)$ is that its degree be the same as the relative degree of $W_p(s)$ and that it be a Hurwitz polynomial and $\bar{L}(s) = L(s)(s+a)$ where $L(s)$ is of degree (n^*-1) and $a > 0$.

From the representation of the plant above, and following, in an exactly analogous manner the developments in [10], we are motivated to represent an observer as shown in Figure 4.

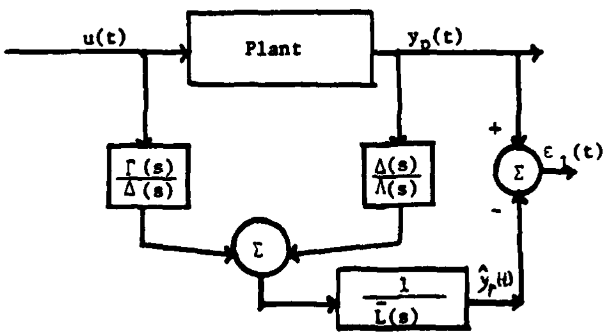

FIGURE 4

In Figure 4, the block representing the plant is realized in the nonminimal representation shown in Figure 3b and (the parameters in) the polynomials $\Gamma(s)$ and $\Delta(s)$ are adjusted using input and output data, so that $\Gamma \to \Gamma^*$ and $\Delta \to \Delta^*$ asymptotically.

The above representation of the plant and observer motivate the basic structure for the model reference controller, which is derived in an exactly analogous manner as in [1]. Figure 5 is a schematic diagram of the controller.

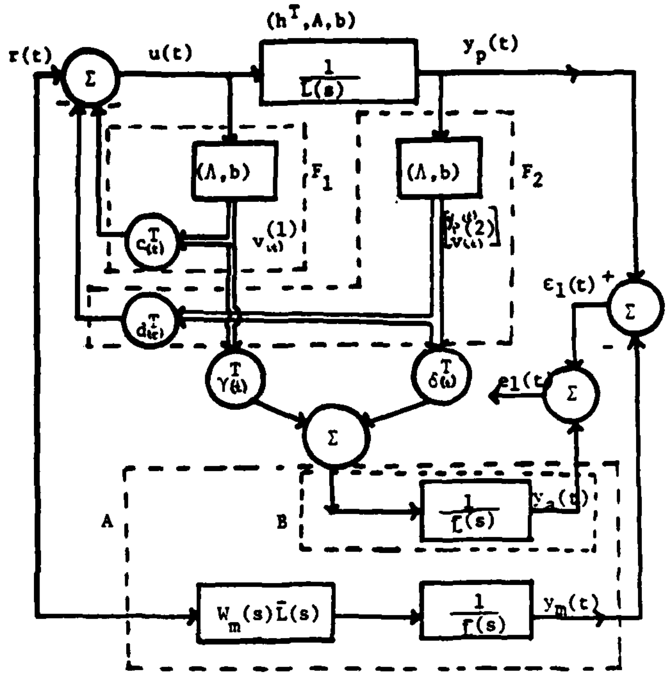

FIGURE 5

Since it can be shown as in [1] that the $(nx1)$ vectors $c^T = \gamma^T$ and the $((n+1)x1)$ vectors $d^T = \delta^T$ the input to $1/\bar{L}(s)$ in block B is 0 in this case. However, block B will be seen later to account for the augmentation to the output error, when deg $(\bar{L}(s)) \geq 2$.

Note: The above diagram corresponds to the case where the gain k_p of the plant is known and can, therefore, without loss of generality, be assumed that $k_p = k_m = 1$. When k_p is unknown, the basic structure remains the same; an extra adjustable parameter c_0 multiplies the reference input to the plant and k_p multiplies $1/\bar{L}(s)$ in its forward path, as shown in dotted circles on Figure 5. If $\Lambda(s) = \bar{L}(s)p_m(s)$, the controller in Figure 5 can match any transfer function $k_m \frac{p_m(s)}{q_m(s)}$. (The proof follows from Corollary 2 after substituting for $\Lambda(s)$, both for k_p known and unknown.)

Note 2: The reference model is incorporated into the scheme of Figure 5 as shown, because we ultimately want to focus attention on the transfer function $\frac{1}{\bar{L}(s)}$ which can be shown [21] to describe the reduced state error dynamics.

THIS ... PRACTICABLE
FROM COPY ...

The state equations for the state variable filters F_1 and F_2 are described by

$$\left.\begin{aligned}\dot{v}^{(1)} &= \Lambda v^{(1)} + bu \\ \omega^{(1)} &= c^T v^{(1)}\end{aligned}\right\} F_1$$
$$\left.\begin{aligned}\dot{v}^{(2)} &= \Lambda v^{(2)} + by_p \\ \omega^{(2)} &= d_o y_p + d^T v^{(2)}\end{aligned}\right\} F_2 \qquad (13)$$

where Λ is an $(n\mathrm{x}n)$ stable matrix (in companion form, usually), $b^T = [0, 0, \ldots, 1)$, $c^T = (c_1, c_2 \ldots, c_n)$ and $d^T = (d_o, d_1 \ldots d_n)$

Defining

$$\bar{\theta}^T(t) \triangleq [c_o(t), \theta^T(t)] \triangleq (c_o(t), c_1(t), \ldots, c_n(t), d_o(t), d_1(t), \ldots d_n(t)); \bar{\omega}^T(t) \triangleq [r(t), \omega^T(t)] \triangleq [r(t), v^{(1)T}(t), y_p(t), v^{(2)T}(t)]$$

the input to the plant, as shown in Figure 5 is

$$u(t) \triangleq \bar{\theta}^T(t)\bar{\omega}(t)$$

and, using the modified model of Section II,

$$u(t) = \bar{\theta}^T(t)\bar{\omega}(t) - \alpha\varepsilon_1(t)\bar{\omega}^T(t)\Gamma\bar{\omega}(t) \qquad \Gamma = \Gamma^T > 0 \quad \alpha > 0$$

When the gain of the plant is known, w.l.o.g. $k_p = k_m = 1$ and $c_o = 0$.

$$\therefore \quad u(t) \triangleq \theta^T(t)\omega(t) - \alpha\varepsilon_1(t)\omega^T(t)\Gamma\omega(t)$$

We shall examine this case first.

(i) $k_p = k_m = 1$ (known)

Following the results of [10] (also summarized by the Lemma and Corollaries in the beginning of this section), a constant control parameter vector θ^* exists such that, if $\theta(t) \equiv \theta^*$ the transfer function of the plant together with the controller matches that of the model exactly. If $\varepsilon(t)$ represents the state error between comparable representations of model and plant, the error differential equations are:

$$\dot{\varepsilon}(t) = A_c\varepsilon(t) + b_c[\phi^T(t)\omega(t) - \alpha\varepsilon_1(t)\omega^T(t)\Gamma\omega(t)]$$
$$\varepsilon_1(t) = h_c^T\varepsilon(t) \qquad (14)$$

where $W_e(s) = h_c^T(sI - A_c)^{-1}b_c = W_m(s)$ and

$$\phi(t) = \theta(t) - \theta^*$$

is the parameter error vector.

a. $n^* = 1$

If the relative degree of the plant is equal to one, w.l.o.g. the model transfer function can be assumed positive real [10]. Hence, according to the results in Section II, the adaptive laws

$$\dot{\phi}(t) = -\varepsilon_1(t)\omega(t)$$

assure the boundedness of $\phi(t)$ and $\varepsilon(t)$. Since the states of the model are bounded, boundedness of the plant states follows and further analysis is identical to that in Section II, for $u(t)$ bounded. ($u(t) = \omega(t)$ here).

It is shown in [21] that the error equations (14) can be expressed in reduced form as

$$\dot{\bar{\varepsilon}}(t) = A\bar{\varepsilon}(t) + b[\phi^T(t)\omega(t) - \alpha\bar{\varepsilon}_1(t)\omega^T(t)\Gamma\omega(t)] \qquad (15)$$
$$\bar{\varepsilon}_1(t) = h^T\bar{\varepsilon}(t) \qquad b^T = (0, \cdot, \cdot, 1) \quad h^T = (1, 0, \cdot, C)$$

and that $\bar{\varepsilon}_1(t)$ and $\varepsilon_1(t)$ are equivalent. A is an $(n^* \times n^*)$ stable matrix and $W_{\bar{\varepsilon}}(s) \triangleq h^T(sI-A)^{-1}b = 1/\bar{L}(s)$, $\omega(t)$, $\phi(t)$ as defined before. Note that with $n^* = 1$, (15) is a scalar differential equation and hence its transfer function is always SPR. The form of equations (15) is particularly useful in the analysis of the controller in Figure 5.

b. $n^* \geq 2$.

If the relative degree $n^* \geq 2$, $W_{\bar{\varepsilon}}(s)$ is a second or higher order (n^*order) transfer function, so that it is not SPR. However, since $\bar{L}(s)$ is known, and arbitrary, a Hurwitz polynonial $L^*(s) = \lambda L(s)$ or $L^*(s) = \lambda\bar{L}(s)$ $\lambda > 0$, can be defined such that $L^*(s)$ $W_{\bar{\varepsilon}}(s) = L^*(s)\bar{L}^{-1}(s)$ is SPR. If every parameter $\theta_i(t)$ in the controller in Figure 5 is replaced by an operator $P_L^*[\theta_i(t)] = L^*(s)\,\theta_i L^{*-1}(s)$, the adaptive laws can be derived according to the error model in Section II as $\dot{\theta}(t) = \Gamma e_1\zeta(t)$, where $\zeta(t) = L^{*-1}(s)I\omega(t)$. However, if the controller is to be differentiator free, such a procedure is not possible through the plant, and more specifically, through the transfer function $\bar{L}^{-1}(s)$ in its forward path. Following then the approach in [10], corresponding to every feedback signal $\theta_i(t)\omega_i(t)$ to the plant, a signal $[\theta_i(t) - P_L^*[\theta_i(t)]]\omega_i(t)$ is realized through the transfer function $\bar{L}^{-1}(s)$ in box B which now forms part of the "augmented model." Hence the positive reality of the error transfer function is preserved, and so are the adaptive laws thus derived. The augmented error controller is shown in Figure 6. The quantities in dotted circles should be ignored (set equal to 1) for the case where k_p is known.

Note that the particular representation of the reference model in Figure 6 is for mathematical convenience only, so as to derive the error equations in the reduced form given by (15). $y_\alpha(t)$ is the augmentation to the model output (block A of Figure 5) and

$$y_\alpha(t) = W_{\bar{\varepsilon}}(s)L^*(s)[(L^{*-1}(s)\theta(t) - \theta(t)L^{*-1}(s))^T\omega(t) + \alpha e_1(t)\zeta^T(t)\Gamma\zeta(t)] \qquad (16)$$

The total output of the model $y(t)$ is the sum of the desired output $y_m(t)$ and the augmented output $y_\alpha(t)$; the augmented error $e_1(t)$ is defined as

$$e_1(t) \triangleq y_p(t) - y(t) = \bar{\varepsilon}_1(t) - y_\alpha(t) \qquad (17)$$

and given by

$$e_1(t) = W_{\bar{\varepsilon}}(s)L^*(s)[\phi^T(t)\zeta(t) - \alpha e_1(t)\zeta^T(t)\Gamma\zeta(t)] \qquad (18)$$

(18) has the same form as (14) with $\omega(t)$ and $\varepsilon_1(t)$ replaced by $\zeta(t)$ and $e_1(t)$ respectively. Also $W_{\bar{\varepsilon}}(s) \triangleq \bar{L}^{-1}(s)$ throughout this section

(ii) k_p unknown

This is the case shown in Figure 6, with the quantities in dotted circles included. Following the analysis in [3], the augmented error equation is now given by

THIS PAGE ... PRACTICABLE
FROM COPY ...

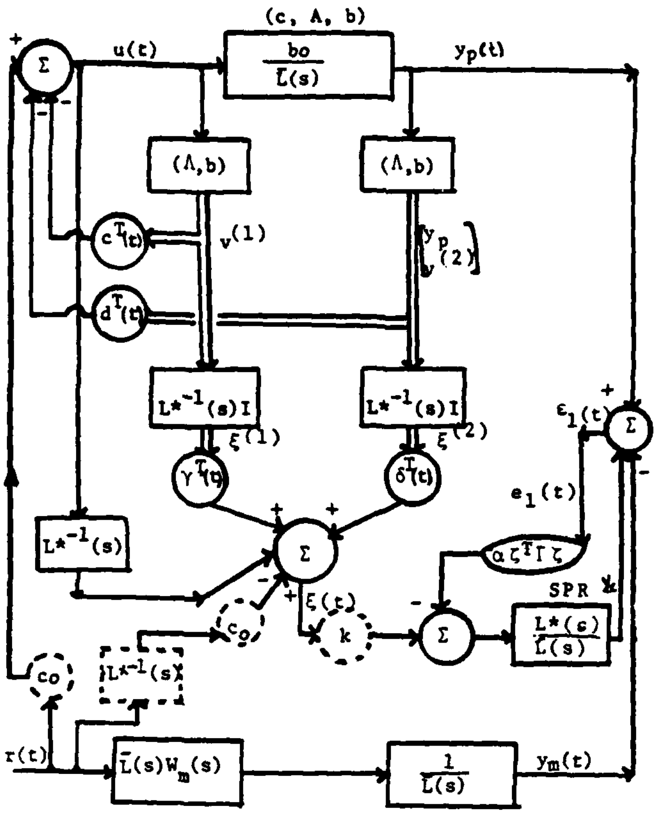

FIGURE 6

$$e_1(t) = \frac{k_p}{k_m}[W_{\bar{\xi}}(s)L^*(s)]\left\{\bar{\phi}^T(t)\bar{\zeta}(t) + \psi(t)\bar{\zeta}(t)\right\} - $$

$$- \alpha e_1(t)\bar{\zeta}(t)^T \Gamma \zeta(t) \qquad (19)$$

and the adaptive laws are

$$\dot{\bar{\phi}}(t) = - \Gamma e_1(t)\bar{\zeta}(t)$$

$$\dot{\Psi}(t) = -\gamma e_1(t)\bar{\bar{\xi}}(t) \qquad (20)$$

where

$$\bar{\zeta}(t) = L^{*-1}(s)I\bar{\omega}(t); \; \bar{\xi}(t) = [L^{*-1}(s)\bar{\phi}(t) - \bar{\phi}(t)L^{*-1}(s)]\bar{\omega}^T(t)$$

$$\text{and } \Psi(t) = 1 - \frac{k_m}{k_p}\psi_1(t) \text{ with } \Psi_1(t) \to \frac{k_p}{k_m}$$

The stability problem that arises is the same in both cases (i) and (ii), with $n^* \geq 2$. In a schematic form, the modified conjecture for $\bar{k}_p$ known in [3,10] is represented as shown in Figure 7.

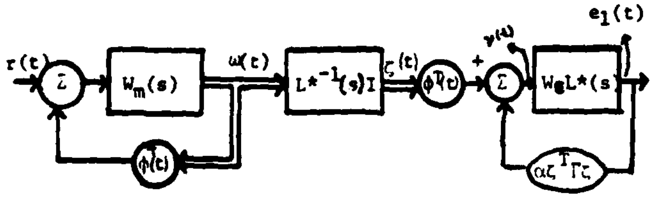

FIGURE 7

Notice that the error transfer function in Figure 7 is at most of first order if $L^*(s) = \lambda L(s) = \frac{\lambda}{s+a}$. This is a generalization to the model considered in [3,10].

V OTHER ADAPTIVE ALGORITHMS

The special parameterization used for the plant and the controller in the preceding section resulted in a very simple structure for the adaptive system and its associated error equations. The model suggested in IV is the simplest controller realization so far and at the same time provides a very precise exposition of the functions of the most essential features - i.e., relative degree, positive reality - in the design of stable adaptive control systems. By choosing different representations for the transfer function $T_{n^*}(s) = \bar{\omega}$ in the forward path of the plant - whose relative degree has to be n^*, same as the plant's - various controllers can be derived. This choice, in turn, affects the "realization" of the SPR transfer function W_e that describes the error dynamics in the transient state of the adaptation process. We mention a few representative algorithms below:

1. Monopoli's Scheme [9]

$$T_{n^*}(s) = p_m(s)/q_m(s) = W_m(s)$$

$$W_e(s) = p_m(s)/q_m(s) \cdot L(s) = W_m(s)L(s),$$

where $L(s)$ is chosen such that $W_e(s)$ is SPR. Contol laws are as follow from use of the error model in Section II.

2. Narendra and Valavani's Schemes: (Both Direct and Indirect Control) [1,10]

$$T_{n^*}(s) = p_m(s)/q_m(s) = W_m(s)$$

$W_e(s) = W_m(s)L(s)$, where $L(s)$ is chosen as in 1.

3. Bénéjean's Scheme [22]

Choices of transfer functions are identidal to Monopoli's.

4. Narendra and Lin's Discrete Control Scheme [23]

[23] is the discrete version of [1,10] in 2 above, and therefore the transfer functions are the discrete-time analogs of those in 2.

5. Goodwin, Ramadge and Caines's Scheme [5]

The algorithms here represent the only adaptive controllers listed so far that were not designed using a stability approach, but are rather projection algorithms - two of them (one each for direct and indirect control) and a third one which is a recursive least squares algorithm, all in discrete-time. Since they are all similar, we mention just one of them here.

GRC Projection Algorithm II

a. Discrete-time

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{\gamma_k \xi_k (y_k - \xi_k^T \hat{\theta}_{k-1})}{1 + \xi_k^T \xi_k} \longrightarrow$$

$$\hat{\theta}_k = \hat{\theta}_{k-1} + \frac{\alpha_k \zeta_k \varepsilon_{1_k}}{1 + \zeta_k^T \zeta_k} = \hat{\theta}_{k-1} + \alpha_k \zeta_k e_{1_k}$$

where ε_{1_k} and e_{1_k} are the output and augmented errors respectively, and ζ_k is the "delayed" (filtered) state vector, all defined in an analog manner as in Section IV.

$$e_{1_k} = \frac{\varepsilon_{1_k}}{1 + \zeta_k^T \zeta_k}$$

b. Continuous-time

Though the authors in [5] described discrete time algorithms only, the continuous time analog of their algorithm above, can be expressed in terms of the general model in IV, with

$$T_n^*(s) = \frac{1}{L(s)}$$

$W_e(s) = L^*(s)\bar{L}^{-1}(s)$, where $L^*(s) = \lambda\bar{L}(s)$. Hence, the only difference from the model in IV is in realizing the augmented error equation, which is now given by

$$e_1(t) = \phi^T(t)\zeta(t)/[\lambda + \alpha\zeta^T(t)\Gamma\zeta(t)] \qquad (22)$$

where $\phi(t)$ is the parameter error vector, $\zeta(t)$ as defined in Section IV and everything else the same as in the model of section IV. (There is no basic difference from the discrete algorithm described in (21), if the scaling factor Γ is included in (22). This is only to make the error equation analogous in form to that described by equation (18)).

VI OUTLINE OF PROOF OF ASYMPTOTIC STABILITY FOR THE CLASS OF CONTROLLERS IN SECTION IV

In this section we present the key ideas in the proof of asymptotic stability of the model described in IV. The nature of the problem is identical to that resolved in [3], the only difference being in the description of the augmented error. Hence, the augmented error is described by a first order differential equation (when $L^*(s) = \lambda L(s)$), or by an algebraic equation (when $L^*(s) = \lambda\bar{L}(s)$). The arguments used here, however, are analogous - if not identical - to those in [3]. The interested reader is referred to [3] or [21] for a rigorous and complete proof. We merely highlight here the important points in its evolution. We list in the appendix key definitions and lemmas in the construction of the stability arguments. We again refer to [3] for a complete discussion and proofs.

A. Statement of the Stability Problem

Using the modified error model in Section II, it is easy to show that e_1, ϕ in equations (17) and (18), or alternatively, e_1, ϕ and Ψ in (19) and (20) are bounded. Moreover, it also follows that $e(t)$, $e_1(t)\zeta(t)$, $\dot{\phi}(t)\varepsilon L^2$. The stability problem is then to show that the plant feedback loop with $\phi(t)$ adjusted according to equation (20) and $\dot{\phi}(t)\varepsilon L^2$ is stable in the large.

B. Proof of Stability

(i) k_p known.

We only need to show that $y_p(t)$ is uniformly bounded. Uniform boundedness of all the signals in the loop follows by Lemma 4.

Considering the reduced error model, we can write

$$y_p(t) = W_m(s)r(t) + \bar{L}^{-1}(s)\phi^T(t)\omega(t) \qquad (23)$$

where $W_m(s)r(t)$ is a uniformly bounded signal. The second term in the RHS of equation (23) can be expressed as

$$\bar{L}^{-1}(s)\phi^T(t)\omega(t) = [\bar{L}^{-1}(s)L^*(s)][L^{*-1}(s)\phi^T(t)L^*(s)]\zeta(t)$$

By lemmas 4 and 5

$$\bar{L}^{-1}(s)\phi^T(t)\omega(t) = [\bar{L}^{-1}(s)L^*(s)]\,\phi^T(t)\zeta(t) + o[\sup_{t\geq\tau}|y_p(\tau)|]$$

Further, since by (18)

$$\phi^T(t)\zeta(t) = \nu(t) + \alpha e_1(t)\zeta^T(t)\Gamma\zeta(t)$$

where $\nu(t) \triangleq \phi^T(t)\zeta(t) - \alpha e_1(t)\zeta^T(t)\Gamma\zeta(t)$ is the input to the error equation

$$\phi^T(t)\zeta(t) = \nu(t) + \alpha\dot{\phi}^T(t)\zeta(t)$$

where

$$[\bar{L}^{-1}(s)L^*(s)]\nu(t) = e_1(t)$$

and

$[\bar{L}^{-1}(s)L^*(s)](\alpha\dot{\phi}^T(t)\zeta(t)) = o[\sup_{t\geq\tau}|y_p(\tau)|]$ by Lemmas 3 and 4 and by $\dot{\phi}\varepsilon L^2$.

$$\therefore\; y_p(t) = W_m(s)r(t) + e_1(t) + o[\sup_{t\geq\tau}|y_p(\tau)|]$$

is bounded, since $e_1(t)$ is bounded from (18), for any choice of $L^*(s)$. $\therefore$ The plant feedback loop is stable in the large.

(ii) k_p unknown

In this case,

$$y_p(t) = [\frac{k_p}{k_m}W_m(s)]\;[\frac{k_m}{k_p}r(t) + \bar{\phi}^T(t)\bar{\omega}(t)] \rightarrow$$

$$y_p(t) = W_m(s)r(t) + [\frac{k_p}{k_m}\bar{L}^{-1}(s)L^*(s)][L^{*-1}(s)\bar{\phi}^T L^*(s)]\bar{\zeta}(t)$$

Analogously as in (1)

$$\frac{k_p}{k_m}\bar{L}^{-1}(s)\bar{\phi}^T(t)\bar{\omega}(t) = [\frac{k_p}{k_m}\bar{L}^{-1}(s)L^*(s)]\,\{\bar{\phi}^T(t)\bar{\zeta}(t) + o[\sup_{t\geq\tau}|y_p(\tau)|]\}$$

From Figure 6

$$\bar{\phi}^T(t)\bar{\zeta}(t) = \bar{\nu}(t) + \alpha\dot{\bar{\phi}}^T(t)\bar{\zeta}(t) - \Psi(t)\bar{\xi}(t)$$

But $\bar{\xi}(t) = [L^{*-1}(s)\bar{\phi}^T(t) - \bar{\phi}^T(t)L^{*-1}(s)]\bar{\omega}(t) =$

$$L^{*-1}(s)\bar{\phi}^T(t)L^*(s)\bar{\zeta}(t) - \bar{\phi}^T(t)\bar{\zeta}(t) = [\sup_{t\geq\tau}|y_p(\tau)|]$$

And since $\Psi(t)$ is uniformly bounded, again we conclude that $y_p(t)$ is also bounded.

(iii) discrete-time model

For the discrete-time case, with $\nu(t)$ replaced by $\nu(k)$ it follows immediately [3] that $\nu(k) \to 0$, $\Delta\phi(k) \to 0$ and hence stability follows trivially.

Since we established that $\omega(t)$ is bounded, then $\zeta(t)$, the input to the error model is bounded, and from the analysis in Section II,

$$\lim_{t\to\infty} \dot{V}(t) = \lim_{t\to\infty} e(t) = \lim_{t\to\infty} \dot{\phi}(t) = 0$$

$\therefore$ the augmented input signal into the error model

$$[L^{*-1}(s)\phi^T(t) - \phi^T(t)L^{*-1}(s)]\omega(t) \text{ and } \alpha e_1(t)\zeta^T(t)\Gamma\bar{\zeta}(t)$$

also tend to zero as $t \to \infty$

$\therefore$ $e_1(t) = y_p(t) - y_m(t) \to 0$ as $t \to \infty$

VII GENERAL COMMENTS

(i) The model presented in Section IV represents a very simple controller structure requiring a minimal number of integrations. This translates into faster response and error dynamics and hence the potential for applications to real world problems. The particular structure described shows clearly the role of the relative degree of the plant in determining the underlying speed of adaptation. This is suggestive and is to be compared with the sensitivity models as in [24] used for optimization.

(ii) The transfer function $\bar{L}^{-1}(s)$ is arbitrary and hence the designer can choose it to improve any one of relevant quantities such as error response of the adaptive system, avoiding to excite unmodeled system dynamics, filtering out disturbances, etc.. These questions have to be investigated thoroughly and systematically in light of this new model. The designer still has no control over the convergence rates of the adaptive control algorithms but this new representation of the adaptive system seems promising. Current research efforts [25] are directed systematically toward understanding closed loop adaptive processes and defining analytically the domains in which they evolve, in terms of relevant design quantities, so that optimization could then take place.

(iii) The fact that the error dynamics are in effect governed by $\bar{L}^{-1}(s)$, which can be chosen arbitrarily, could have further implications on the robustness properties of the adaptive system.

(iv) We observe that positive reality of the error equations is ultimately only connected with the adjustment of the parameters and can be viewed separately from the rest of the plant-model adaptive configuration.

(v) Proof of the original conjecture formulated in [10] suddenly seems important and more tractable in light of the new representation and as encompassing a larger class of adaptive problems, without the limitation that ϕ has to be adjusted so that $\dot{\phi}\epsilon L^2$.

VIII CONCLUSION

Stability questions that arise in the deterministic adaptive control of SISO systems and methods for their analysis have been reviewed, with special emphasis given to the "augmented error" systems. "Augmented error" systems arise when the relative degree of the plant transfer function is greater than or equal to two. The question of asymptotic stability of this type of an error system, even in the deterministic case, remained unresolved for a long-time. Recently, various proofs were proposed for specific adaptive controller designs. In Section IV of the paper, a general model for a controller structure is proposed, using a specific parameterization. Other existing algorithms are seen to be special cases of this general model, regardless of which approach was used as a basis for the design. An outline of a proof for the model in Section IV was given, which is the most general proof for asymptotic stability, that holds for all the existing deterministic adaptive controllers. However, no specific close form theoretical results are available for the convergence rates of these algorithms or their behavior (in a global sense) in the presence of disturbances. Such questions are the subject of current research.

REFERENCES

1. K.S. Narendra and L.S. Valavani, "Direct and Indirect Model Reference Adaptive Control," Automatica, vol. 15, November 1979, pp. 653-664.

2. B. Egardt, "Unification of Some Continuous - Time Adaptive Control Schemes," IEEE Trans. Aut. Contr., vol. AC-24, no. 4, August 1979, pp. 588-592.

3. K.S. Narendra, Y.H. Lin, L.S. Valavani, "Stable Adaptive Controller Design, Part II: Proof of Stability," S & IS Report No. 7904, Yale University, April 1979. Forthcoming in IEEE Trans. Aut. Contr.

4. A.S. Morse, "Global Stability of Parameter - Adaptive Control Systems," S & IS Report No. 7902R, Yale University, March 1979.

5. G.C. Goodwin, P.J. Ramadge and P.E Caines, "Discrete Time Multivariable Adaptive Control," Harvard University Technical Report, November 1978.

6. L. Ljung, "On Positive Real Transfer Functions and the Convergence of Some Recursive Schemes," IEEE Trans. Aut. Contr., vol. AC-22, no. 4, August 1977, pp. 539-551.

7. L. Ljung, "Analysis of Recursive Stochastic Algorithms," ibid., pp. 551-575.

8. K.S. Narendra and L.S. Valavani, "Stable Adaptive Observers and Controllers," Proc. IEEE, vol. 64, no. 8, August 1976, pp. 1198-1208.

9. R.V. Monopoli, "Model Reference Adaptive Control with an Augmented Error Signal," IEEE Trans. Aut. Contr., vol. AC-19, October 1974, pp. 474-484.

10. K.S. Narendra and L.S. Valavani, "Stable Adaptive Controller Design-Direct Control, IEEE Trans. Aut. Contr., vol. AC-23, August 1978, pp. 570-583.

11. A. Feuer and A.S. Morse, "Adaptive Control of a Single-Input Single-Output Linear System," IEEE Trans. Aut. Contr., vol. AC-23, August 1978, pp. 557-570.

12. H. Elliott and W.A. Wolovich, "Parameter Adaptive Identification and Control," in Proc. 1978 Conf. on Decision and Control, San Diego, CA, January 1979, pp. 602-607.

13. G. Kreisselmeier, "Algebraic Separation in Realizing a Linear State Feedback Control Law by Means of an Adaptive Observer," IEEE Trans. Aut. Contr., vol. AC-25, no. 2, April 1980, pp. 238-243.

14. L.S. Valavani, "Stable Adaptive Control of Dynamical Systems," Ph.D. dissertation, ch. 3, Yale University, May 1978.

15. A.P. Morgan and K.S. Narendra, "On the Stability of Non-Autonomous Differential Equations $\dot{x} = [A + B(t)]x$, with Skew Symmetric Matrix B(t)," SIAM J. Control and Optimization, vol. 15, no. 1, January 1977, pp. 163-176.

16. A.P. Morgan and K.S. Narendra, "On the Uniform Asymptotic Stability of Certain Linear Time-Varying Equations with Unbounded Coefficients," Yale University Technical Report, Dept. of Engineering and Applied Science, 1978.

17. K.S. Narendra and L.S. Valavani, "A Comparison of Lyapunov and Hyperstability Approaches to Adaptive Control of Continuous Systems," IEEE Trans. Aut. Contr. vol. AC-25, no. 2, April 1980, pp. 243-347.

18. V.M. Popov, Hyperstability of Control Systems, Springer-Verlag, New York, 1973.

19. I.D. Landau, "Hyperstability Concepts and Their Application to Discrete Control Systems," in Proc. JACC, Stanford, CA, August, 1972.

20. I.D. Landau, "A Generalization of the Hyperstability Conditions for Model Reference Adaptive Systems," IEEE Trans. Aut. Contr., vol. AC-17, pp. 246-247, 1972.

21. L.S. Valavani, "A Simple Representative Adaptive Controller Structure for a Class of Augmented Error Systems," submitted to IEEE Trans. Aut. Contr.

22. R. Bénéjean, La Commande Adaptive à Modèle de Référence Evolutif. Université Scientifique et Médicale de Grenoble, France, 1977.

23. K.S. Narendra and Y.H. Lin, "Stable Adaptive Identification and Control of Discrete Systems," Yale University Technical Report, 1978.

24. J.B. Cruz, ed., System Sensitivity Analysis, Dowden, Hutchinson and Ross, Inc., Stroudsburg, Pa., 1973.

25. C. Rohrs, L. Valavani and M. Athans, "Convergence Studies of Adaptive Control Algorithms - Part I: Analysis," to be presented at the 1980 CDC, Albuquerque, NM.

APPENDIX

Definition 1:

$$L_e^\infty = \{f: R^+ \to R \mid \sup_{t \geq \tau} |f(\tau)| < \infty, \ \forall t \in R^+\}$$

Definition 2: Let $x(t), y(t) \in L_e^\infty$. Let $\beta(t)$ be a continuous function such that $\beta(t) \to 0$ as $t \to \infty$. If $y(t) = \beta(t)x(t)$, we denote

$$y(t) = o[x(t)]$$

Remark 1:

(i) If $y(t) = o[y(t)]$, the $y(t) \to 0$ as $t \to \infty$

(ii) If $|y(t)| = o[\sup|y(\tau)|]$, then $y(t) \to 0$ as $t \to \infty$

(iii) If $|y(t)| = x(t) + o[\sup_{t \geq \tau}|y(\tau)|]$ and $x(t)$ is uniformly bounded, then $y(t)$ is also uniformly bounded.

Definition 3: Let $x(t), y(t) \in L_e^\infty$. If there exists a constant $M > 0$ such that $|y(t)| \leq M|x(t)|$, then we denote

$$y(t) = O[x(t)]$$

Remark 2: If the input to a linear exponentially stable system is $x(t) \in L_e^\infty$ and the corresponding output is $y(t)$, then

$$y(t) = O[\sup_{t \geq \tau} |x(\tau)|]$$

Definition 4: Let $x(t), y(t) \in L_e^\infty$. If $y(t) = O[x(t)]$ and $x(t) = O[y(t)]$, then we say that $x(t)$ and $y(t)$ are equivalent and denote this by

$$x(t) \sim y(t)$$

Definition 5: Let $x(t), y(t) \in L_e^\infty$. If $\sup|y(\tau)| \sim \sup|x(\tau)|$, we say that $x(t)$ and $y(t)$ grow at the same rate.

Remark 4:

(i) If $x(t), y(t) \in L_e^\infty$, one and only one of the following three conditions can hold:

$$\sup_{t \geq \tau}|x(\tau)| \sim \sup_{t \geq \tau}|y(\tau)|, \quad \sup_{t \geq \tau}|y(\tau)| = o[\sup_{t \geq \tau}|x(\tau)|],$$

$$\text{or} \quad \sup_{t \geq \tau}|x(\tau)| = o[\sup_{t \geq \tau}|y(\tau)|]$$

(ii) $$\left\{\sup_{t \geq \tau}|x(\tau)| = o[\sup_{t \geq \tau}|y(\tau)|] \text{ or } \sup_{t \geq \tau}|x(\tau)| \sim \sup_{t \geq \tau}|y(\tau)|\right\} \Rightarrow \left\{\sup_{t \geq \tau}|x(\tau)| = O[\sup_{t \geq \tau}|y(\tau)|]\right\}$$

(iii) Let an n-dimensional vector $x(t) \in L_e^\infty$ be unbounded, then there exists at least a component $x_{i_0}(t)$ such the

$$\sup_{t \geq \tau}|x_{i_0}(\tau)| \sim \sup_{t \geq \tau}\|x(\tau)\|$$